Step by Step Procedure of ABC

Consider the optimization problem as follows:

\[ \text{Minimize } f(x) = x_1^2 + x_2^2, \quad -5 \leq x_1, x_2 \leq 5 \]

Control Parameters of ABC Algorithm are set as;
- Colony size, CS = 6
- Limit for scout, L = (CS*D)/2 = 6

and dimension of the problem, D = 2

First, we initialize the positions of 3 food sources (CS/2) of employed bees, randomly using uniform distribution in the range (-5, 5).

\[ x = \begin{bmatrix} 1.4112 & -2.5644 \\ 0.4756 & 1.4338 \\ -0.1824 & -1.0323 \end{bmatrix} \]

\[ f(x) \text{ values are;} \]
- 8.5678
- 2.2820
- 1.0990

Fitness function:  
\[ \text{fit}_i = \begin{cases} \frac{1}{1 + f_i} & \text{if } f_i \geq 0 \\ \frac{1 + \text{abs}(f_i)}{1 + f_i} & \text{if } f_i < 0 \end{cases} \]

Initial fitness vector is:
- 0.1045
- 0.3047
- 0.4764

Maximum fitness value is 0.4764, the quality of the best food source.

Cycle=1

//Employed bees phase

- 1\textsuperscript{st} employed bee

\[ v_{i,j} = x_{i,j} + \Phi_f (x_{i,j} - x_{k,j}) \text{ with this formula, produce a new solution.} \]
\[ k=1 \quad \text{//k is a random selected index.} \]
\[ j=0 \quad \text{//j is a random selected index.} \]
\[ \Phi = 0.8050 \quad // \Phi \text{ is randomly produced number in the range } [-1, 1]. \]
\[ \nu_0 = \begin{pmatrix} 2.1644 \\ -2.5644 \end{pmatrix} \]

- Calculate \( f(\nu_0) \) and the fitness of \( \nu_0 \).

\[ f(\nu_0) = 11.2610 \text{ and the fitness value is } 0.0816. \]

- Apply greedy selection between \( x_0 \) and \( \nu_0 \)

\[ 0.0816 < 0.1045, \text{ the solution } 0 \text{ couldn’t be improved, increase its trial counter.} \]

- 2\textsuperscript{nd} employed bee

\[ \nu_{ij} = x_{ij} + \Phi_{ij} (x_{ij} - x_{kj}) \quad \text{with this formula produce a new solution.} \]
\[ k = 2 \quad //k \text{ is a random selected solution in the neighborhood of } i. \]
\[ j = 1 \quad //j \text{ is a random selected dimension of the problem.} \]
\[ \Phi = 0.0762 \quad // \Phi \text{ is randomly produced number in the range } [-1, 1]. \]
\[ \nu_1 = \begin{pmatrix} 0.4756 \\ 1.6217 \end{pmatrix} \]

- Calculate \( f(\nu_1) \) and the fitness of \( \nu_1 \).

\[ f(\nu_1) = 2.8560 \text{ and the fitness value is } 0.2593. \]

- Apply greedy selection between \( x_1 \) and \( \nu_1 \)

\[ 0.2593 < 0.3047, \text{ the solution } 1 \text{ couldn’t be improved, increase its trial counter.} \]

- 3\textsuperscript{rd} employed bee

\[ \nu_{ij} = x_{ij} + \Phi_{ij} (x_{ij} - x_{kj}) \quad \text{with this formula produce a new solution.} \]
\[ k = 0 \quad //k \text{ is a random selected solution in the neighborhood of } i. \]
\[ j = 0 \quad //j \text{ is a random selected dimension of the problem.} \]
\[ \Phi = -0.0671 \quad // \Phi \text{ is randomly produced number in the range } [-1, 1]. \]
\[ \nu_2 = \begin{pmatrix} -0.0754 \\ -1.0323 \end{pmatrix} \]

- Calculate \( f(\nu_2) \) and the fitness of \( \nu_2 \).

\[ f(\nu_2) = 1.0714 \text{ and the fitness value is } 0.4828. \]

- Apply greedy selection between \( x_2 \) and \( \nu_2 \).
0.4828 > 0.4764, the solution 2 was improved, set its trial counter as 0 and replace the solution \( x_2 \) with \( \nu_2 \).

\[
x = \\
1.4112 -2.5644  \\
0.4756 1.4338  \\
-0.0754 -1.0323
\]

\( f(x) \) values are:

\[
\begin{align*}
8.5678 \\
2.2820 \\
1.0714
\end{align*}
\]

fitness vector is:

\[
\begin{align*}
0.1045 \\
0.3047 \\
0.4828
\end{align*}
\]

//Calculate the probability values \( p \) for the solutions \( x \) by means of their fitness values by using the formula:

\[
p_i = \frac{f_{i}}{\sum_{i=1}^{CS} f_{i}}
\]

\[
p = \\
0.1172 \\
0.3416 \\
0.5412
\]

//Onlooker bees phase
//Produce new solutions \( \nu_i \) for the onlookers from the solutions \( x_i \) selected depending on \( p_i \) and evaluate them.

- 1\textsuperscript{st} onlooker bee
  - \( \nu_2 \) =
    \[
    \begin{align*}
    -0.0754 & \quad -2.2520
    \end{align*}
    \]
  - Calculate \( f(\nu_2) \) and the fitness of \( \nu_2 \).
    
    \[
    f(\nu_2) = 5.0772 \text{ and the fitness value is } 0.1645.
    \]
  - Apply greedy selection between \( x_2 \) and \( \nu_2 \)
0.1645 < 0.4828, the solution 2 couldn’t be improved, increase its trial counter.

- 2nd onlooker bee
  - \( i = 1 \)
    - \( \upsilon_1 = \begin{bmatrix} 0.1722 \\ 1.4338 \end{bmatrix} \)
    - Calculate \( f(\upsilon_1) \) and the fitness of \( \upsilon_1 \).
      - \( f(\upsilon_1) = 2.0855 \) and the fitness value is 0.3241.
    - Apply greedy selection between \( x_1 \) and \( \upsilon_1 \)
      - 0.3241 > 0.3047, the solution 1 was improved, set its trial counter as 0 and replace the solution \( x_1 \) with \( \upsilon_1 \).

- \( x = \begin{bmatrix} 1.4112 \\ -2.5644 \\ 0.1722 \\ 1.4338 \\ -0.0754 \\ -1.0323 \end{bmatrix} \)
  - \( f(x) \) values are;
    - 8.5678
    - 2.0855
    - 1.0714

  - fitness vector is:
    - 0.1045
    - 0.3241
    - 0.4828

- 3rd onlooker bee
  - \( i = 2 \)
    - \( \upsilon_2 = \begin{bmatrix} 0.0348 \\ -1.0323 \end{bmatrix} \)
    - Calculate \( f(\upsilon_2) \) and the fitness of \( \upsilon_2 \).
      - \( f(\upsilon_2) = 1.0669 \) and the fitness value is 0.4838.
    - Apply greedy selection between \( x_2 \) and \( \upsilon_2 \)
0.4838 > 0.4828, the solution 2 was improved, set its trial counter as 0 and replace the solution $x_2$ with $v_2$.

\[
x =
\begin{bmatrix}
1.4112 & -2.5644 \\
0.1722 & 1.4338 \\
0.0348 & -1.0323
\end{bmatrix}
\]

\[f(x)\text{ values are;}
\begin{align*}
&8.5678 \\
&2.0855 \\
&1.0669
\end{align*}
\]

fitness vector is:
\begin{align*}
&0.1045 \\
&0.3241 \\
&0.4838
\end{align*}

//Memorize best
Best =
\begin{align*}
&0.0348 \\
&-1.0323
\end{align*}

//Scout bee phase
TrialCounter =
\begin{align*}
&1 \\
&0 \\
&0
\end{align*}

//There is no abandoned solution since L = 6
//If there is an abandoned solution (the solution of which the trial counter value is higher than L = 6); generate a new solution randomly to replace with the abandoned one.
Cycle = Cycle + 1

The procedure is continued until the termination criterion is attained.